Physics 251

Lecture VII - Lift - An Introduction

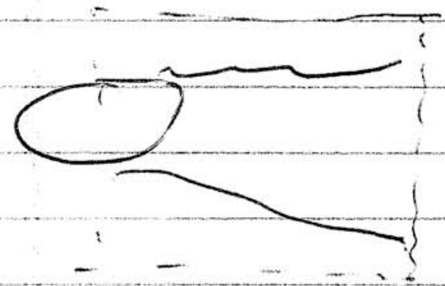
→ Have developed drag, wakes, etc. from:

- boundary layers, separation

and

- force differential

i.e



$$F_d = (\Delta P_{tot})A$$

Now, can just as easily compute force differential cross-section cross-stream

$$\Delta (P_{tot} A)_\perp = P_1 A_1 - P_2 A_2 = F_{Lift}$$

Now, a bit more carefully:

$$F_i = \int da_k \, \Pi_{i,k}$$

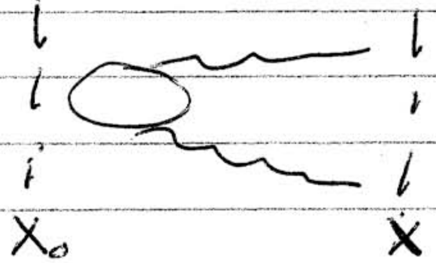
ith component force on body

stress tensor

$$\Pi_{ik} = (p_0 + p')\delta_{i,k} + \rho(U_i + v_i)(U_k + v_k)$$

allow flow in arb. direction

So



$\underline{U} = U\hat{x}$

$$F_i = \left(\int_{X_0} da_i - \int_{X} da_i\right) \cdot (p'\delta_{ix} + \rho U v_i)$$

flow deviation from pot. flow

$F_x = F_d \Leftrightarrow$ wake

$F_y = F_L \Leftrightarrow$ also wake, ... but force $\perp$ $\Rightarrow$ Lift

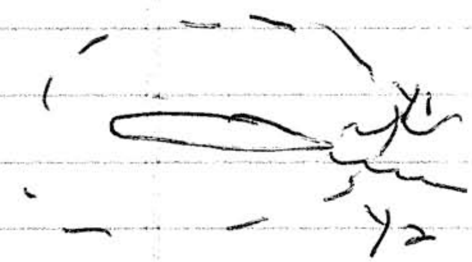
Lift is wake origin effect!

i.e. ⇒ pure potential flow ⇒ no wake
⇒ no drag, no lift

→ wake ⇒ lift requires friction-induced separation

So,

$$F_y = -\rho U \iint v_y \, dy \, dz \qquad (\Pi_{xy})$$



~ wake thin (*)
[~ U_y comparable in, out (Q balance), but larger domain out]

(*)

⇒

$$\int_{-\infty}^{+\infty} dy \, v_y \cong \int_{y_1}^{\infty} v_y \, dy + \int_{-\infty}^{y_2} v_y \, dy$$

$y_2 \sim y_1 + \delta$

as outside wake ⇒ flow is potential!

$\underline{v} = \nabla \phi$

$$\int_{-\infty}^{+\infty} dy \, v_y \cong \int_{y_1}^{\infty} \frac{\partial \phi}{\partial y} dy + \int_{-\infty}^{y_2} \frac{\partial \phi}{\partial y} dy$$

$\phi(\pm\infty) = 0$

$\int_{-\infty}^{+\infty} dy\, v_y \cong \phi_2 - \phi_1$

$$\boxed{F_y \cong -\rho U \int dz\,(\phi_2 - \phi_1)}$$

but $\int_C \underline{\nabla}\phi \cdot d\underline{l} = \phi_2 - \phi_1$

C = contour encl. body

As this wake: wake ~ $\Delta y\, V_y$

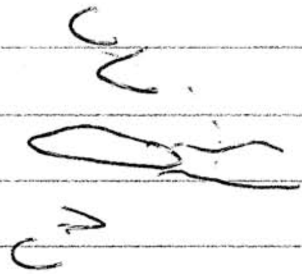
$$\int_C \underline{\nabla}\phi \cdot d\underline{l} \cong \oint_{C+\text{wake}} \underline{V} \cdot d\underline{l} \equiv \Gamma$$

Γ: circulation of wing

$\cong \phi_2 - \phi_1$

↳ depends on cut

⇒ $$\boxed{F_y = -\rho U \int \Gamma \, dz}$$



→ Zhukovskii's Theorem

$$F_L/_{\text{Length}} \cong -\rho U \Gamma$$

Lift ⇔ circulation

→ $U > 0 \Rightarrow \Gamma < 0$ for $F_y > 0$

→ sense of circulation for lift.

→ What is in C_L?

$$F_y = -\rho U \Gamma l_z$$

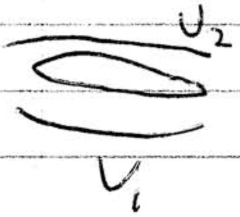
$$F_y = C_L \frac{\rho}{2} U^2 A$$

C_L ⇔ contains info on structure of circulation and wing shape, orientation.

clearly $\boxed{\Gamma \Leftrightarrow C_L U l_y}$

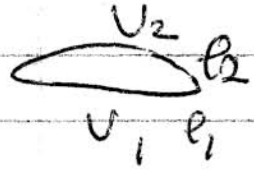
estimate of circulation!

→ Fairy Tales about Lift

– 

$V_2 > V_1 \Rightarrow P_2 < P_1$
(Bernoulli)

⇒ OK → not to the point, though. Circulation is fundamental

– 

$V_2 > V_1$ as $\ell_2 > \ell_1$

NO → particles starting at leading edge do not meet at trailing edge.

→ More

– $\oint \underline{v} \cdot d\ell \neq 0 = \int d\underline{a} \cdot \underline{\omega} \neq 0$

↓
cross-section area of wing

d.e. Wing
region containing vortical flow.

– non-zero circulation ⇔ wake

i.e. $\Gamma = \oint \underline{v} \cdot d\underline{\ell}$
$= \int_C \underline{v} \cdot d\ell + \Delta y \, v_y$

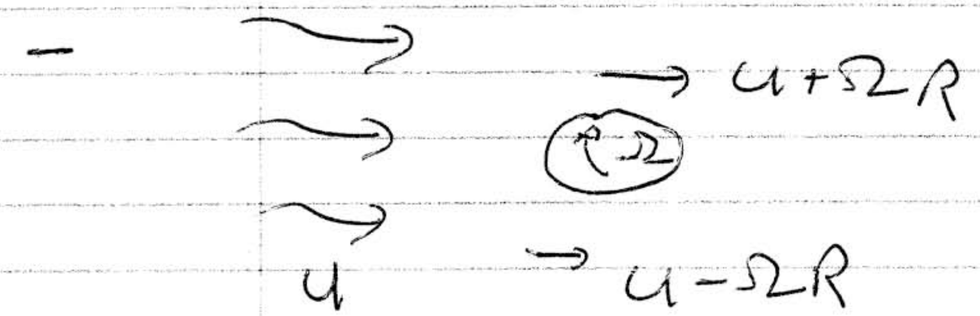
$V_y \Delta y \to 0$, thin wake $\Rightarrow$ justifies cut.

$\Gamma = \phi_2 - \phi_1$

- Lift $\Rightarrow$ broken up-down symmetry required.

$\to$ Lift $\Leftrightarrow$ Magnus Force.

- can have lift via Magnus Force, without circulation



higher speed $\to$ low pressure

lower speed $\to$ high pressure

$$\Delta p = \rho\left[(u+\Omega R)^2 - (u-\Omega R)^2\right]/2$$

$$\cong 2\rho u \Omega R$$

$\Delta p > 0 \Rightarrow$ Lift!

→ Summary of Wakes

– wakes ⇔ small viscosity changes flow everywhere

i.e. $\Delta Q = 0$, $Q_{\text{pot flow}} \sim VA \Rightarrow$

$V \sim 1/r^2$ global

$\phi \sim 1/r$

– How?

Vorticity produced in BL, transported out.

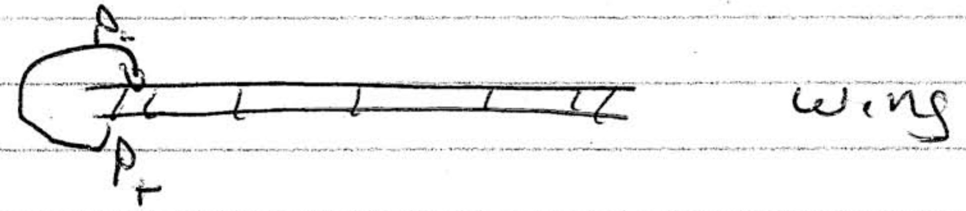
– Why? Viscosity changes boundary conditions.

⟷ singular perturbation.

– What does wake of wing look like?

→ wake is thin sheet behind wing

→ ∞ wing → $w \sim \sqrt{x}$ (show!)

→ but 

⇒ sheet 'rolls up' at edges.